NOTATION

y, axis of symmetry of the sprayer; r, distance from the axis of symmetry; δ, thickness of liquid film; s, coordinate measured along the surface of the sprayer; n, coordinate measured along a normal to the sprayer surface; φ, azimuthal coordinate; u, v, w, corresponding components of the velocity vector; p, pressure; F, centrifugal force; R, radius of curvature of the sprayer surface; $\Delta\Sigma$, element of the sprayer surface; θ, angle of inclination of the surface to the y axis; ρ, liquid density; l, L, characteristic lengths; h, Lamé coefficient; Eu, Euler number; A, α, λ, similarity parameters; ψ, stream function.

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EFFECTIVE TRANSPORT COEFFICIENTS IN A DISPERSE MEDIUM WITH ELLIPSOIDAL INCLUSIONS

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Expressions are obtained for the steady-state conductivity tensor for moderately concentrated heterogeneous materials with ellipsoidal inclusions.

If the linear dimensions of the mean temperature or concentration fields in a heterogeneous medium (consisting of a homogeneous matrix with discrete inclusions distributed in it) are significantly larger than the characteristic dimensions of the inclusions, then heat or mass transport is naturally described in terms of the continuum approximation. In this case it is sufficient to introduce effective thermal conductivities or diffusion coefficients for the medium as a whole [1, 2].

The determination of these effective coefficients for a medium with spherical inclusions has been considered in a number of papers, but the number of papers devoted to the analogous problem for a medium with nonspherical inclusions is quite small. A dilute dispersion of nonspherical inclusions was considered in [3]. A moderately concentrated dispersion of spheroidal inclusions was studied in [4, 5] in the dipole approximation (where the contribution of each inclusion to the mean field is replaced by that of a point dipole at the center of the given inclusion). In the present paper the general methods of [2] are used to analyze the properties of a heterogeneous material with ellipsoidal inclusions. The spatial distribution of the ellipsoids is assumed to be random and their orientation is assumed to obey a given statistical distribution law which is identical for all points of space. Then the material is macroscopically homogeneous, although it is not necessarily isotropic. We note that this theory is important not only in the description of materials with inclusions, but also as a model for the analysis of transport processes in isotropic and anisotropic polycrystalline media of more complicated structure [6, 7].

Statement of the Problem. In an anisotropic heterogeneous medium the relation between the mean heat flux and the gradient of the mean temperature has the form

$$\mathbf{q} = -\boldsymbol{\lambda}\cdot\nabla\tau, \tag{1}$$

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where λ is a symmetric tensor of the second rank. In the special case when the sizes and shapes of all the ellipsoids are identical, this tensor can be found using the method of [2] from the relations [5]

$$\lambda - \lambda_0 I = (\lambda_1 - \lambda_0)\rho\mathbf{v},$$
$$\mathbf{v}\cdot\mathbf{E}(\mathbf{R}) = \frac{1}{V}\int_{\Omega}\int_{\mathbf{r}\in V}\nabla\tau_{\Omega}^{*}(\mathbf{R}+\mathbf{r}|\mathbf{R})\,d\mathbf{r}\varphi(\Omega)\,d\Omega, \tag{2}$$

where Ω denotes the set of variables determining the orientation of an ellipsoid, and $\tau_{\Omega}^{*}(\mathbf{R}+\mathbf{r}|\mathbf{R})$ denotes the temperature inside an ellipsoid whose center is at the point R and whose orientation is characterized by Ω, averaged over all physically allowed positions and orientations of all of the other inclusions. The integration with respect to $d\mathbf{r}$ goes over the volume V of an isolated (test) ellipsoid, and the orientational distribution function $\varphi(\Omega)$ is normalized to unity.

If the volume concentration of inclusions is not large, then the test ellipsoid can be considered, approximately, to be embedded in a fictitious medium, whose properties are the same as those of the original heterogeneous medium. This model of a moderately concentrated medium corresponds to neglecting the fact that the ellipsoids cannot overlap [2, 4, 5]. In this case the field τ_{Ω}^{*} can be determined from the solution of the pollowing problem for the test ellipsoid:

$$\nabla\cdot(\lambda\cdot\nabla\tau') = 0,\ \mathbf{r}\notin V;\ \Delta\tau_{\Omega}^{*} = 0,\ \mathbf{r}\in V;$$
$$\tau'\to 0,\ r\to\infty;\ \tau_{\Omega}^{*}<\infty,\ r=0; \tag{3}$$
$$\mathbf{E}\cdot\mathbf{r}+\tau' = \tau_{\Omega}^{*},\ \mathbf{n}\cdot\lambda\cdot(E+\nabla\tau') = \lambda_1\mathbf{n}\cdot\nabla\tau_{\Omega}^{*},\ \mathbf{r}\in S.$$

Here S and V are the surface and volume of an ellipsoid of a given orientation, whose center is chosen as the origin of coordinates ($\mathbf{R} = 0$); $\mathbf{n}$ is a unit vector normal to S; $\tau'(\mathbf{r})$ is interpreted as the perturbation of the linear mean temperature due to the test ellipsoid; the vector $\mathbf{E}$ is a constant vector defined at the center of the ellipsoid. The solution of the boundary-value problem (3) can be used to find the function $\tau_{\Omega}^{*}(\mathbf{R}+\mathbf{r}|\mathbf{R}) = \tau_{\Omega}^{*}(\mathbf{r})$, which depends on the components λ as well as the parameters. Using this function in the integral of (2), we obtain a system of three transcendental equations for the three unknown principal values of the tensor λ. In general the principal axes of this tensor do not coincide with the principal axes of the test ellipsoid, whose directions are characterized by the orientational variables Ω.

Temperature Field inside the Test Ellipsoid. The complete solution of (3), which is not difficult to obtain in ellipsoidal coordinates, is very complicated. However, the complete solution is not of primary interest in the context of the present paper; rather we need to find only the mean temperature gradient $E^{*}(\Omega) = \nabla\tau_{\Omega}^{*}$ inside an ellipsoid of a given orientation. Therefore we employ certain well-known results directly, without solving the boundary-value problem in detail.

We introduce the Cartesian coordinates x, y, z taken along the principal axes of the tensor λ and consider first a test ellipsoid whose principal axes are oriented along the coordinate axes. The semiaxes of the ellipsoid are denoted by a, b, and c. A uniform gradient of the mean temperature far from the ellipsoid can be represented as the vector sum $E_x\mathbf{e}_x+E_y\mathbf{e}_y+E_z\mathbf{e}_z$, where the $\mathbf{e}_i$ (i = x, y, z) are unit vectors. We consider separately the effect of each term in this sum on the field inside the ellipsoid. From symmetry considerations it is obvious that the field inside the ellipsoid excited by the external field $E_x\mathbf{e}_x\cdot\mathbf{r}$ is such that its gradient is parallel to $\mathbf{e}_x$, i.e., it can be represented in the form $E_x^{*}\mathbf{e}_x$. In order to use the results of [8] we perform a scaling transformation of the coordinates, thereby transforming the operator $\nabla\cdot(\lambda\cdot\nabla)$ outside the ellipsoid into the operator $\lambda_x\Delta$. It is important that the mean temperature gradient remain unchanged in the x direction after the scaling transformation is performed. Hence the relation between the corresponding heat fluxes and gradients must remain invariant to the transformation. The transformation has the form

$$x' = x,\ y' = \gamma_y y,\ z' = \gamma_z z,\ \gamma_{y,z} = \lambda_x/\lambda_{y,z}.$$

As a result of the transformation the ellipsoid is deformed and its new semiaxes are $a'=a,\ b'=\gamma_y b,\ c'=\gamma_z c$. Following the reasoning of [8], it can be shown that the vector field E_x^{*} is uniform and we thereby obtain

$$E_x^* = \frac{\lambda_x}{\lambda_x + (\lambda_1 - \lambda_x) n_x} E_x,$$

where n_x is the corresponding eigenvalue of the depolarization tensor of the deformed ellipsoid, and depends not only on a, b, c, but also on the principal values λ_i of the tensor λ.

In a completely analogous way we can calculate the temperature gradient inside an ellipsoid excited by the external fields $E_y \mathbf{e}_y \cdot r$ and $E_z \mathbf{e}_z \cdot r$. Here we use transformations which leave the scale of the coordinates y or z unchanged, respectively. We then obtain the final expression for the temperature gradient inside the ellipsoid

$$\mathbf{E}^* = \frac{\lambda_x E_x}{\lambda_x + (\lambda_1 - \lambda_x) n_x} \mathbf{e}_x + \frac{\lambda_y E_y}{\lambda_y + (\lambda_1 - \lambda_y) n_y} \mathbf{e}_y + \frac{\lambda_z E_z}{\lambda_z + (\lambda_1 - \lambda_z) n_z} \mathbf{e}_z, \tag{4}$$

and n_x, n_y, n_z are the eigenvalues of the depolarization tensors of different (but similar in shape) ellipsoids. Since these quantities depend only on the ratios of the semiaxes, and not on the volume of the ellipsoid, they can be determined for a single ellipsoid with principal semiaxes a', b', c'.

The generalization to the case of an ellipsoid whose principal axes X, Y, Z are oriented arbitrarily to the principal axes x, y, z of the tensor λ is trivial. It is sufficient to represent the external temperature field in the form $E_X \mathbf{e}_X + E_Y \mathbf{e}_Y + E_Z \mathbf{e}_Z$ and calculate the components of the temperature gradient inside the ellipsoid due to the three terms of this sum, using scaling transformations of the coordinates x, y, z which leave the scale unchanged in the direction of the axes X, Y, or Z, respectively. Then we again obtain a formula of the type (4), in which x, y, z is replaced by X, Y, Z, and the quantities n_i (i = X, Y, Z) depend in addition on the variables Ω (since the principal semiaxes of the ellipsoids deformed by the scaling transformations depend on Ω).

Using these results and (2), we can determine a system of equations for the principal values of effective thermal conductivity tensor in (1), for a disperse medium with an arbitrary orientational distribution function of inclusions. We consider in more detail the cases where all inclusions are oriented in the same direction, and where the orientation of the inclusions is random. In the first case the medium is anisotropic and its principal axes coincide with the principal axes of the ellipsoidal inclusions; in the second case the medium is isotropic.

Material with Identically Oriented Inclusions. Substituting (4) into (2), we obtain a system of three equations for λ_x, λ_y, λ_z:

$$\frac{\lambda_i - \lambda_0}{\lambda_1 - \lambda_0} = \frac{\rho \lambda_i}{\lambda_i + (\lambda_1 - \lambda_i) n_i}, \quad i = x, y, z, \tag{5}$$

and the λ_i-dependent eigenvalues of the depolarization tensor are given in the form

$$n_i = \frac{1}{2} \int_0^\infty \frac{a'b'c'dx}{(a_i'^2 + x)[(a'^2 + x)(b'^2 + x)(c'^2 + x)]^{1/2}}, \quad a_i' = \begin{Bmatrix} a' \\ b' \\ c' \end{Bmatrix}. \tag{6}$$

For a dilute medium we can assume, approximately, that $\lambda_i = \lambda_0$. In this case λ_i on the right side of (5) can be replaced by λ_0, and n_i by the quantities n_i^0, referred to the undeformed ellipsoid with principal semiaxes a, b, c. Then we obtain the result corresponding to the theory of [3]

$$\lambda_i = \lambda_0 \left[1 + \frac{\rho (\varkappa - 1)}{1 + (\varkappa - 1) n_i^\circ} \right], \quad \varkappa = \frac{\lambda_1}{\lambda_0},$$

where the n_i^0 are obtained using formulas of the type (6).

Equation (5) simplifies considerably in the limiting cases of perfectly conducting ($\kappa \to \infty$) and nonconducting ($\kappa = 0$) inclusions. If we assume that the ratios of the semiaxes b/a and c/a are fixed then

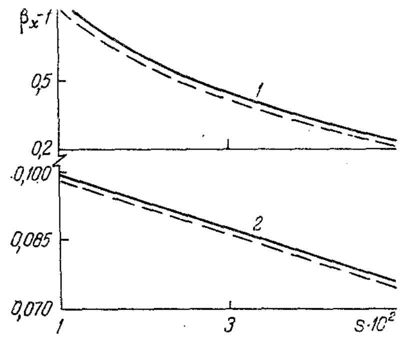

Fig. 1

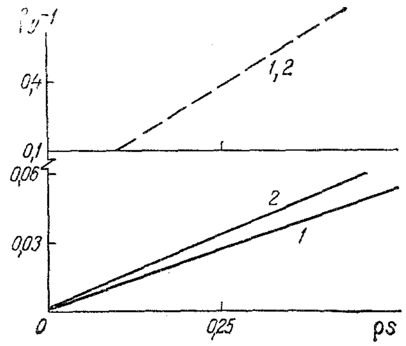

Fig. 2

Fig. 1. Relative longitudinal conductivity of a medium with highly conducting needlelike inclusions as a function of s = b/a for ρ = 0.01; solid curves: the solution (12); dashed curves: from (13); 1) log κ = 2; 2) log κ = 3.

Fig. 2. Relative transverse conductivity of a medium with highly conducting disklike inclusions as a function of ρs with κ/s = 0.5 (1) and 50 (2). Solid curves: the solution (15), dashed curves: (16).

$$\lambda_i = \begin{cases} \lambda_0 \left(1 - \dfrac{\rho}{n_i}\right)^{-1}, & \varkappa \to \infty; \\ \lambda_0 \left(1 - \dfrac{\rho}{1-n_i}\right), & \varkappa = 0. \end{cases} \tag{7}$$

We consider in more detail a medium with inclusions in the form of spheroids, i.e., ellipsoids of rotation (b = c). In this case $\lambda_y = \lambda_z$, the system of equations for λ_x and λ_y coincide in form with (5), and in place of (6) we can write [8]:

$$n_x = \begin{cases} \dfrac{1-e^2}{2e^3}\left(\ln\dfrac{1+e}{1-e} - 2e\right), & \dfrac{b}{a}\sqrt{\dfrac{\lambda_x}{\lambda_y}} < 1; \\ \dfrac{1+e^2}{e^3}(e - \operatorname{arctg} e), & \dfrac{b}{a}\sqrt{\dfrac{\lambda_x}{\lambda_y}} > 1; \end{cases}$$
$$n_y = n_z = \frac{1}{2}(1 - n_x), \quad e = \left| 1 - \left(\frac{b}{a}\right)^2 \frac{\lambda_x}{\lambda_y} \right|^{1/2}. \tag{8}$$

For a medium with spherical inclusions $n_x = n_y = n_z = 1/3$ and from (5) we obtain the well-known equation for the effective scalar thermal conductivity of a disperse medium [9]

$$\frac{\lambda - \lambda_0}{\lambda_1 - \lambda_0} = \frac{3\rho\lambda}{\lambda_1 + 2\lambda},$$

which is valid approximately for a medium with a moderate concentration of the dispersed phase. We note that this equation differs from the analogous result obtained in the dipole approximation [10].

For a medium with needlelike inclusions (the limit $s = b/a \to 0$ for finite κ) we have $n_x \to 0$, $n_y = n_z \to 1/2$ and it follows from (5) that

$$\lambda_x \to \lambda_0 [1 + \rho(\varkappa - 1)],$$
$$\lambda_y = \lambda_z \to (\lambda_0/2)\{[(1-2\rho)^2(\varkappa-1)^2 + 4\varkappa]^{1/2} - (1-2\rho)(\varkappa - 1)\}, \tag{9}$$

which is the same as the result of [11] for heterogeneous materials with parallel cylindrical fibers.

For a medium with disklike inclusions (the limit $s = b/a \to \infty$ for finite κ) we have $n_x \to 1$, $n_y = n_z \to 0$ and it follows from (5) that

$$\lambda_x \to \lambda_0 [1 - \rho (1 - \varkappa^{-1})]^{-1}, \ \lambda_y = \lambda_z \to \lambda_0 [1 + \rho (\varkappa - 1)], \tag{10}$$

which is an obvious result for a layered medium.

Equations (9) and (10) can become invalid if κ or κ^{-1} goes to zero or infinitely more rapidly than s or s^{-1}. We consider these singular cases in more detail. For a medium with highly conducting, elongated spheroidal inclusions $s \ll 1$ and $\varkappa \gg 1$. Taking into account only the principal terms in the small-parameter expansion of (8), for such inclusions we obtain, approximately

$$n_x \approx m = \frac{\alpha}{2} \ln \frac{1}{\alpha} \ll 1, \ n_y = n_z \approx \frac{1}{2}, \ \alpha = s^2 \frac{\lambda_x}{\lambda_y}.$$

Using this in (5), we see that the transverse thermal conductivity is given by the second equation in (9) for all $\varkappa$, as before, and we obtain the following equation for the relative longitudinal thermal conductivity in the limit $\kappa \gg 1$ (here $\beta_i = \lambda_i/\lambda_0$):

$$\beta_x - 1 \approx \frac{\rho \varkappa \beta_x}{\beta_x + \varkappa m}, \ \beta_y \approx \frac{1}{1 - 2\rho}, \ m = \frac{\alpha}{2} \ln \frac{1}{\alpha}, \tag{11}$$

where the formula for β_y follows from (9) in the limit $\kappa \to \infty$. We see from (11) that the limiting relations for β_x corresponding to equations (7) and (9) are obtained for $\varkappa m \gg \epsilon \varkappa$ and $\varkappa m \ll \beta_x$, respectively. Therefore, it is not difficult to determine the region of approximate validity of these relations in the parameter space. In general, it follows from (11) that

$$\beta_x \approx 1 + \rho \varkappa \left\{ 1 + \frac{1 - 2\rho}{2} \varkappa s^2 \left[\ln \frac{1}{(1 - 2\rho) s^2} - \ln \beta_x \right] \right\}^{-1}. \tag{12}$$

If $\rho \ll 1$ and also $s^{-2} \gg \beta_x$, then with logarithmic accuracy we have

$$\beta_x \approx 1 + \rho \varkappa \left(1 + \varkappa s^2 \ln \frac{1}{s} \right)^{-1}. \tag{13}$$

The solution of equation (12) and the result (13) are illustrated in Fig. 1.

We consider now a medium with highly conducting disklike inclusions, when $s \gg 1$ and $\kappa \gg 1$. In this case we have from (8)

$$n_x \approx 1 - 2m, \ n_y = n_z \approx m = \frac{\pi}{4 \sqrt{\alpha}} \ll 1, \ \alpha = s^2 \frac{\lambda_x}{\lambda_y}.$$

The longitudinal thermal conductivity is expressed, as before, by the first equation of (10) for all κ, and in place of the second formula for $\kappa \gg 1$ we obtain the following equation for the relative transverse thermal conductivity:

$$\beta_y - 1 \approx \frac{\rho \varkappa \beta_y}{\beta_y + \varkappa m}, \ \beta_x \approx \frac{1}{1 - \rho}, \ m = \frac{\pi}{4 \sqrt{\alpha}}. \tag{14}$$

In the limits $\varkappa m \gg \beta_y$ and $\varkappa m \ll \beta_y$ we then obtain the limiting relations corresponding to (7) and (10). In the general case it follows from (14) that

$$\beta_y \approx 1 + \rho \varkappa \left(1 + \frac{\pi \varkappa}{4s} \frac{\sqrt{1 - \rho}}{\sqrt{\beta_y}} \right)^{-1}. \tag{15}$$

If $\varkappa \ll s\sqrt{\beta_y}$ we have the result which follows from (10); in the opposite case (strong inequality) we have (if $\rho \ll 1$)

$$\beta_y \approx \left[\left(1 + \frac{4 s^2 \rho^2}{\pi^2} \right)^{1/2} + \frac{2 s \rho}{\pi} \right]^2. \tag{16}$$

The quantities β_y as obtained from (15) and (16) are shown in Fig. 2.

The transverse thermal conductivity of a medium with poorly conducting disklike inclusions can be obtained from (10) in the limit $\varkappa \ll 1$, i.e., $\beta_y \approx 1 - \rho$. However, the longitudinal

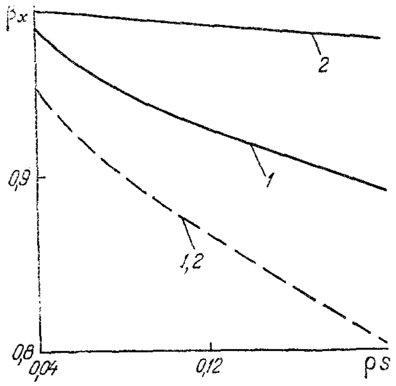

Fig. 3. Relative longitudinal conductivity of a medium with poorly conducting disklike inclusions as a function of ρs from (17) and (18) (solid and dashed curves respectively) for log (κs) = 1 (1) and 0 (2).

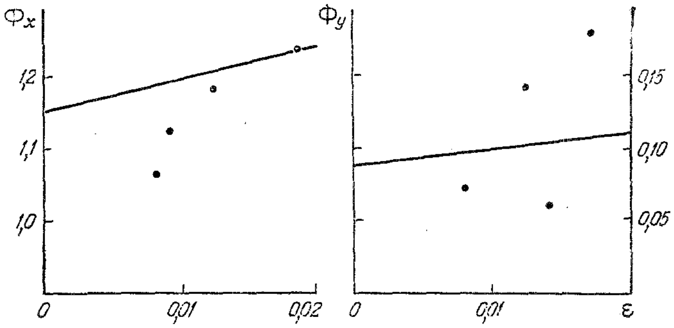

Fig. 4. Dependence of the quantities $\Phi_i=(\lambda_i/\lambda_0-1)/\varepsilon$ $(i=x, y)$ on $\varepsilon=\rho/s^2\ln(2/s)$ for a medium with highly conducting needlelike inclusions for s = 0.0106, $\varkappa \gg 1$. Curves: calculated results from (7); points: the data of [13].

thermal conductivity of such a medium (in a direction perpendicular to the plane of the disks) can be found with the help of the solution of the equation

$$\beta_x \approx 1-\rho s\sqrt{\beta_x}\left(\frac{\varkappa s}{\sqrt{\beta_x}}+\frac{\pi}{4\sqrt{1-\rho}}\right)^{-1} \tag{17}$$

and is given by the formula $\beta_x \approx (1+\rho/\varkappa)^{-1}$, following from (10) only when $\varkappa s \gg \beta_x \lesssim 1$. If we have instead the opposite strong inequality and also $\rho \ll 1$, then from (17) we have

$$\beta_x \approx \left[\left(1+\frac{4s^2\rho^2}{\pi^2}\right)^{1/2}-\frac{2s\rho}{\pi}\right]^2. \tag{18}$$

The solution (17) and equation (18) are illustrated in Fig. 3.

These results suggest that highly nonspherical needlelike or disklike spheroids with a high thermal conductivity can strongly affect the effective longitudinal or transverse thermal conductivities of a heterogeneous medium with identically oriented inclusions, even if their volume concentration ρ is very small. Similarly, the presence of identically oriented nonconducting dislike inclusions can significantly decrease the thermal conductivity in the direction perpendicular to the plane of the disks, even when $\rho \ll 1$. Indeed, it follows from the relations given above that the effective thermal conductivities β_x and β_y are proportional not to ρ itself, but to ρs^2 (this is the volume concentration of spheres in the medium, where the radius of a sphere is equal to the larger of the two semiaxes of the

TABLE 1. Relative Effective Thermal Conductivity of a Moderately Concentrated Medium with Randomly Oriented Spheroidal Inclusions

$\varkappa$	$n_x^\circ=0,1$			$n_x^\circ=0,3$			$n_x^\circ=0,6$		
	ρ								
	0,03	0,07	0,09	0,09	0,15	0,21	0,12	0,18	0,24
0,01	0,97 0,95	0,93 0,90	0,91 0,87	0,89 0,85	0,83 0,79	0,76 0,72	0,84 0,82	0,77 0,74	0,64 0,59
100	1,15 1,15	1,43 1,44	1,62 1,64	1,35 1,36	1,75 1,78	2,46 2,51	1,77 1,78	2,71 2,78	2,79 2,82

Note: The upper rows of numbers are the results from (20); the lower rows are the results from the dipole approximation [4].

spheroid) or to $\rho\varkappa$ or $\rho\varkappa^{-1}$ (for $\varkappa \gg 1$ and $\varkappa \ll 1$, respectively). This fact has been noted earlier [3-5] and has been discussed in detail in [1]. A similar behavior is also characteristic for the effective viscosities of suspensions of needlelike particles undergoing straining flows [12].

The number of experiments measuring the effective conductivities of media with identically oriented inclusions, and subject to sufficiently controlled conditions, is not large. In Fig. 4 we compare the theory developed here and the experimental data of [13]. If we take into account that the experimental data is characterized by large dispersion, and the fact that the inclusions are really not spheroids, but wire fragments, many of which are curved, we see that the correspondence between the theory and experiment can be considered to be satisfactory.

Materials with Randomly Oriented Inclusions. In this case the material is not only macroscopically homogeneous, but also isotropic. Using (4) with $\lambda_x = \lambda_y = \lambda_z = \lambda$ and averaging over all equally probable orientations of the test ellipsoid, we obtain from (5) an equation for λ

$$\frac{\lambda-\lambda_0}{\lambda_1-\lambda_0}=\frac{\rho}{3}\left[\frac{1}{\lambda+(\lambda_1-\lambda)n_x^\circ}+\frac{1}{\lambda+(\lambda_1-\lambda)n_y^\circ}+\frac{1}{\lambda+(\lambda_1-\lambda)n_z^\circ}\right], \tag{19}$$

where n_i^0 are the eigenvalues of the depolarization tensor for the undeformed ellipsoid with semiaxes a, b, c, calculated from formulas analogous to (6). For a medium with spheroidal inclusions ($b = c$) this equation simplifies considerably:

$$\frac{\lambda-\lambda_0}{\lambda_1-\lambda_0}=\frac{\rho}{3}\left[\frac{1}{\lambda+(\lambda_1-\lambda)n_x^\circ}+\frac{4}{2\lambda+(\lambda_1-\lambda)(1-n_x^\circ)}\right]\lambda, \tag{20}$$

where n_x° can be found from (8) with $\lambda_x/\lambda_y = 1$.

From (19) and (20) it is not difficult to obtain the well-known expressions for the effective thermal conductivities of a dilute suspension with ellipsoidal inclusions and of a moderately concentrated material containing needlelike spheroidal particles.

In the limiting cases of needlelike and disklike spheroids, we obtain from (20) equations replacing (9) and (10):

$$\begin{gathered}\lambda\to\frac{\lambda_0}{2}\left\{\left(1-\frac{5}{3}\rho\right)(\varkappa-1)+\left[\left(1-\frac{5}{3}\rho\right)^2(\varkappa-1)^2+4\varkappa\left(1+\frac{\rho}{3}(\varkappa-1)\right)\right]^{1/2}\right\},\ n_x^\circ\to 0,\\ \lambda\to\lambda_0\left[1+\frac{2}{3}\rho(\varkappa-1)\right]\left[1-\frac{\rho}{3}\left(1-\frac{1}{\varkappa}\right)\right]^{-1},\ n_x^\circ\to 1.\end{gathered} \tag{21}$$

In place of (7) we obtain for perfectly conducting and nonconducting spheroids in this case

$$\lambda = \lambda_0 \left[1 + \rho \frac{n_x^\circ + 1/3}{n_x^\circ (1 - n_x^\circ)} \right]^{-1}, \ \varkappa \to \infty,$$
$$\lambda = \lambda_0 \left[1 + \frac{\rho}{3} (\varkappa - 1) \frac{5 - 3n_x^\circ}{1 - n_x^{\circ 2}} \right], \ \varkappa \to 0. \quad (22)$$

Some of the results following from (20) are collected in Table 1 where the corresponding results calculated with the dipole approximation [4] are also shown. We see that overall the dipole approximation is completely satisfactory for computing the effective thermal conductivities of heterogeneous materials.

As in the case of a medium with identically oriented inclusions, a strong dependence of the effective thermal conductivities on the presence in the inclusions, even for very small concentrations, is also characteristic of a medium with randomly oriented inclusions. Since for a disperse medium with spherical particles, the model of a moderately concentrated medium (where the impenetrability of the particles is neglected) leads to satisfactory results up to $\rho \leqslant 0.20$-0.25, in the case considered here, particularly for a random orientation of inclusions, one expects errors at smaller values of the concentration. Similar results were obtained in [11] for fibrous materials as well. Therefore it is of considerable interest to develop the theory further, and take into account the actual distribution of the centers of the ellipsoids in the neighborhood of the test ellipsoid.

According to the model of a moderately concentrated medium, the thermal conductivities for ρ = const depend upon the shapes and orientations of the inclusions, but not on their sizes. In this model the results obtained here can easily be extended to the case when there are inclusions of different shapes in the medium; to handle this problem it is sufficient to average (2) over the variables characterizing the shape of the ellipsoids, as well as over the orientational variable Ω.

In conclusion we emphasize that in view of the equivalence of the mathematical formulation of the various transport problems for a test ellipsoid, the equations given above can be used directly to compute the steady-state effective coefficient of diffusion of the impurities in media with ellipsoidal inclusions, and also the effective electrical conductivities, dielectric permitivities, and magnetic susceptibilities of such materials. Our results for a medium with disklike inclusions can be used to determine the lumped component of the permeability of fissured-porous materials; this is an important problem in applications. In this sense, our results are a significant extension of those of [14], where the components of the effective interstitial permeability tensor were calculated.

NOTATION

a, b, c, lengths of the semiaxes of the ellipsoids; e, defined in (8); $\mathbf{e}$, unit vector; $\mathbf{E}$, mean temperature gradient; $\mathbf{I}$, unit tensor of the second rank; $\mathbf{q}$, flux; $\mathbf{n}$, unit vector normal to the surface of the test particle; m, defined in (11); n_i, principal values of the depolarization tensor of a test ellipsoid; S and V, its surface and volume; s, ratio of the semiaxes of the spheroid; $\beta_i = \lambda_i/\lambda_0$; $\gamma_i = \lambda_i/\lambda_x$; λ, thermal conductivity; ν, tensor defined in (2); ρ, volume concentration of the dispersed phase; τ, mean temperature; Ω, characterizes the orientation of the ellipsoid; $\varphi(\Omega)$, orientational distribution function of the ellipsoids; $\varkappa = \lambda_1/\lambda_0$; τ_Ω^* and τ', temperature inside the test ellipsoid and the perturbation caused by it in the mean temperature; the subscripts 0 and 1 denote quantities referring to the matrix and inclusions, respectively.

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MASS TRANSFER IN A SOLID PARTICLE WITH COMPETING REACTIONS WITH A MULTICOMPONENT GAS MIXTURE

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A macrokinetic model of the transformation of a solid particle, reacting with a multicomponent gas mixture, is constructed for arbitrary ratios between the rates of the mass-transfer stages of the transformation process (sorption, dissolution, and diffusion of the starting and final products).

Processes for working solid dispersed materials with multicomponent gaseous mixtures are widely used in modern technology. In the general case a macrokinetic model of the transformation of a solid particle reacting in the atmosphere of a gaseous mixture must take into account all elementary mass-transfer stages of the reaction: sorption—desorption of reagents and reaction products from both phases and their dissolution and diffusion in the solid particle. Well-known theoretical studies [1, 2] usually presume that there exists one limiting stage of mass transfer, which is insufficient for describing reactions of practical interest. The model of the solid-phase transformation, constructed in [3, 4] and presuming that the rates of several stages are comparable, must be generalized to the case of the interaction of solid spherical particles with gaseous mixtures. Modeling such processes enables the calculation and optimization of different states as well as the intensification of the interaction of solid particles with the gas phase by increasing the partial pressures of gaseous reagents or by changing the composition of the gaseous mixture.

In studying a gas mixture we assume that we have N gaseous reagents and that correspondingly, N gaseous reaction products form. Under the assumptions made in [3, 4], we assume that the chemical reaction involved in the interaction of the solid reagent with each gaseous reagent itself proceeds much more rapidly than the mass transfer processes, and the reaction front separates the region of the starting reagent and the solid product of the reaction. Analogously to [3, 4] we shall formulate the equations of kinetics of all stages of the process.

Defining S_i and $S_i^!$ as the relative fractions of the area of the surface layer filled with

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